

Module 5 – Solutions

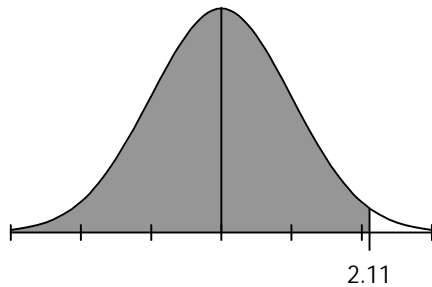
- 5.1 (a) (i) Where only two outcomes are possible, i.e. success or failure.
 (ii) p is the probability of success, q is the probability of failure, n is the number of trials
 r is the number of successes for n trials.

(b) (i) $P(\text{hit}) = \frac{2}{3}$ $P(\text{miss}) = \frac{1}{3}$ $n = 10$
 $P(9 \text{ hits}) = \binom{10}{9} \left(\frac{2}{3}\right)^9 \left(\frac{1}{3}\right)^1 = \frac{10 \times 2^9}{3^{10}} = 0.0867$

(ii) $P(10 \text{ hits}) = \left(\frac{2}{3}\right)^{10} = 0.0173$
 $P(< 9) = 1 - [P(9) + P(10)]$
 $P(< 9) = 1 - [0.0867 + 0.0173]$
 $P(< 9) = 0.896$
 Thus fewer than 9 times \Rightarrow probability = 0.896

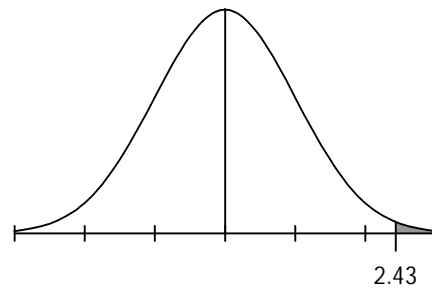
5.2 68%, 95%, 99.7%

5.3 (a)



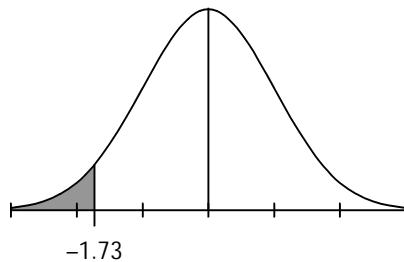
$P(z \leq 2.11) = 0.9826$ (98.26%)

(b)



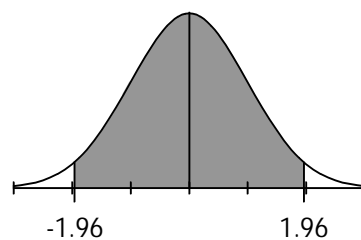
$P(z \geq 2.43) = 1 - P(z \leq 2.43)$
 $P(z \geq 2.43) = 1 - 0.9925$
 $P(z \geq 2.43) = 0.0075$ (0.75%)

(c)



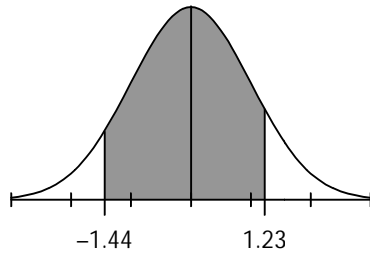
$P(z \leq -1.73) = P(z \geq 1.73)$
 $P(z \leq -1.73) = 1 - P(z \leq 1.73)$
 $P(z \leq -1.73) = 1 - 0.9582$
 $P(z \leq -1.73) = 0.0418$ (4.18%)

(d)



$P(-1.96 \leq z \leq 1.96)$
 $P(-1.96 \leq z) = P(z \geq -1.96) = 1 - P(z \leq -1.96)$
 $P(-1.96 \leq z \leq 1.96) = \text{Area to the left of } 1.96$
 $\quad \quad \quad - \text{Area to the left of } -1.96$
 $P(-1.96 \leq z \leq 1.96) = P(z \leq 1.96) - [1 - P(z \leq 1.96)]$
 $P(-1.96 \leq z \leq 1.96) = 0.9750 - [1 - 0.9750]$
 $P(-1.96 \leq z \leq 1.96) = 0.95$ (95%)

(e)



$$P(-1.44 \leq z \leq 1.23)$$

$$P(-1.44 \leq z) = P(z \geq -1.44) = 1 - P(z \leq 1.44)$$

$$P(-1.44 \leq z \leq 1.23) = \text{Area to the left of } 1.23$$

– Area to the left of -1.44

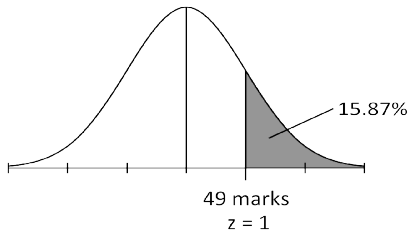
$$P(-1.96 \leq z \leq 1.96) = P(z \leq 1.23) - [1 - P(z \leq 1.44)]$$

$$P(-1.96 \leq z \leq 1.96) = 0.8907 - [1 - 0.9251]$$

$$P(-1.96 \leq z \leq 1.96) = 0.8158 \quad (81.58\%)$$

5.4

$$z = \frac{x - \mu}{\sigma} = \frac{49 - 45}{4} = 1$$



⇒ The probability of scoring above 1 in the standard normal distribution is $1 - 0.8413 = 0.1587$.

The percentage of people scoring above the mean is 50%.

The percentage of people scoring higher than 49 is approx. 16%.

The percentage of people scoring above the mean but lower than 49 is $50 - 16 = 34\%$.

5.5

(a)
$$z = \frac{x - \mu}{\sigma} = \frac{23 - 26}{4} = -0.75$$

Can only look up positive values in tables

$$P(Z < -0.75)$$

$$P(Z > 0.75) = 1 - 0.7734 = 0.2266$$

22.66% chance of getting married younger than 23.

(b)

$$90\% = 0.90 \approx 0.8997 \text{ [closest in tables]}$$

$$z = 1.28$$

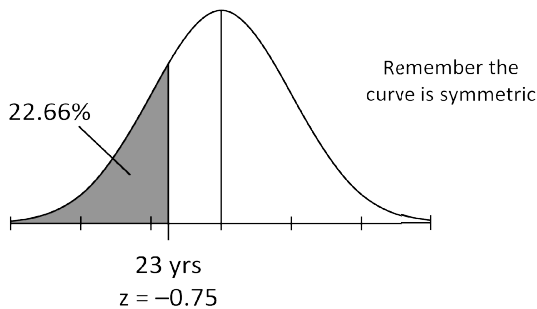
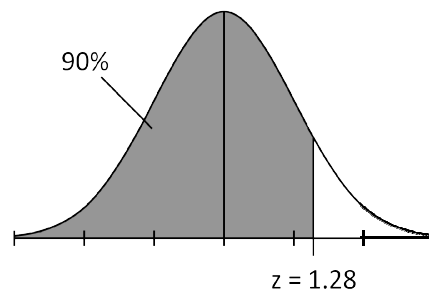
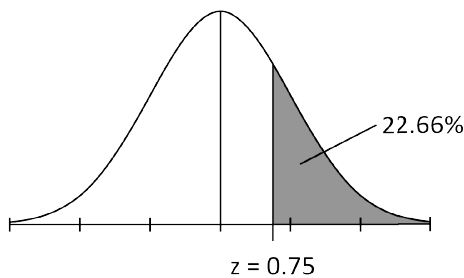
$$z = \frac{x - \mu}{\sigma}$$

$$1.28 = \frac{x - 26}{4}$$

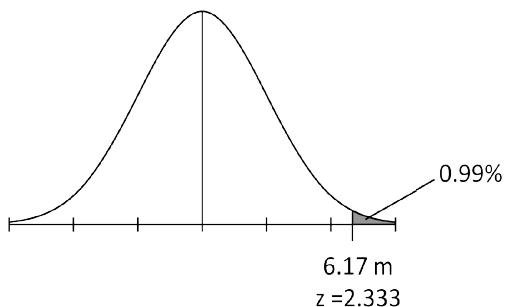
$$(4)(1.28) = x - 26$$

$$5.12 + 26 = x$$

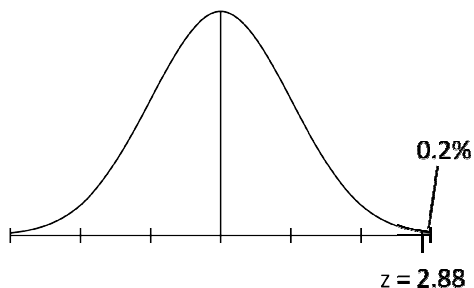
$$31.12 \text{ years} = x$$



5.6 (a) $z = \frac{x - \mu}{\sigma} = \frac{6.17 - 6.1}{0.03} = 2.333$
 $P(Z \leq 2.33) = 0.9901$
 $P(Z > 2.33) = 1 - 0.9901 = 0.0099$
 0.99% chance



(b) 1 in 500 = 0.002 [0.2% of Jumps]
 99.8% $\Rightarrow z = 2.88$
 $z = \frac{x - \mu}{\sigma}$
 $2.88 = \frac{x - 6.1}{0.03}$
 $(2.88)(0.03) = x - 6.1$
 $0.0864 + 6.1 = x$
 $6.186 \text{ m} = x$



- 5.7 (i) 5% = margin of error.
 (ii) Null hypothesis : There is no difference in the attitude of Leinster students to PM.
 According to the results of the survey we fail to accept the null hypothesis as 45% is outside the margin of error of the results for Munster which is from 55% to 65%.